5 SEM TDC MTH M 1

2018

(November)

MATHEMATICS

(Major)

Course : 501

(Logic and Combinatorics, and Analysis—III)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

(A) Logic and Combinatorics

(*Marks* : 35)

1. *(a)* *(i)* Under what condition a sentence is termed as proposition?

(ii) Define a truth function. 1×2=2

(b) *(i)* Find denial of $\sim(\sim P \wedge Q)$. 1

(ii) Let P be 'John will sit' and Q be 'George will wait'. Give verbal sentence which describes each of the following : 1×2=2

(1) $P \vee \sim Q$

(2) $\sim P \wedge \sim Q$

(c) (i) Construct the truth table for $(p \wedge q) \to \sim q$. State whether it is a tautology or not. 2

(ii) Find the truth value of $p \wedge (p \to q)$ using arithmetical representation. 3

2. (a) Define equivalence of statements. 1

(b) State the rules of inferences. 2

(c) Prove that $A \to (C \to B)$, $\sim D \vee A$, $C \models D \to B$. 3

(d) Derive any *one* of the following : 4

(i) Everyone who buys a ticket receives a prize. Therefore, if there is no prize, there nobody buys ticket.

(ii) No human beings are quadrupeds. All women are human beings. Therefore, no woman is quadruped.

3. *(a)* State Vandermonde's identity. 1

(b) How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2 and x_3 are non-negative integers? 2

Or

In an election the number of candidates is one more than the number of vacancies. If a voter can vote in 30 different ways, find the number of candidates.

(c) Define Catalan number. Prove that the nth Catalan number defined from $(1, 0)$ to $(n, n-1)$ is given by

$$\frac{1}{n}C(2n-2,\ n-1)$$ 1+3=4

Or

Define Stirling number. Prove that

$$[x]^n = \sum_{k=1}^{n} U(n,\ k)\,[x]_k$$

where $U(n,\ k) = \frac{\lfloor n}{\lfloor k} C(n-1,\ k-1)$. 4

4. *(a)* Define Ramsay number. 1

(b) Show that if m and n are integers, both greater than 2, then

$$R(m, n) \leq R(m-1, n) + R(m, n-1)$$ 3

Or

How many integers between 500 to 1000 are divisible by 3 or 5?

(c) Define exponential generating function. Find the generating function to count the number of integral solutions $e_1 + e_2 + e_3 = 10$, if for each i, $e_i \geq 0$. 1+3=4

Or

There are 300 boxes with oranges. Each contains no more than x oranges. Find the maximum possible value of x such that 3 boxes contain equal number of oranges. 4

(B) Analysis—III (Complex Analysis)

(*Marks* : 45)

5. *(a)* State the necessary condition for a complex function $f(z)$ to be analytic. 1

(b) Show that an analytic function with constant modulus is constant. 3

Or

For what values of z, the function w defined by $z = \log \rho + i\phi$, where $w = \rho(\cos\phi + i \sin\phi)$, ceases to be analytic?

(c) Find the imaginary part of the analytic function whose real part

$$u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$ 6

Or

Prove that $u = e^{-x}(x\cos y + y\sin y)$ is harmonic. Find its harmonic conjugate.

6. *(a)* Define a rectifiable curve. 1

(b) Find the value of

$$\int_0^{1+i} (x - y + ix^2)\, dz$$

along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$. 4

(c) State and prove Cauchy's theorem. 5

(d) Answer the following (any *one*) : 4

(i) Evaluate

$$\int_C \frac{e^z}{z^2+1}\,dz$$

where C is given by $|z|=2$.

(ii) Evaluate

$$\int_C \frac{z-1}{(z+1)^2(z-2)}\,dz$$

where C is such that $|z-i|=2$.

7. (a) State Laurent's series. 1

(b) Expand $\cos z$ in a Taylor's series about $z=\pi/4$. 3

(c) Expand

$$f(z)=\frac{1}{z(z^2-3z+2)}$$

for the region $1<|z|<2$. 4

Or

Expand e^z in a Taylor's series about $z=0$ and determine the region of convergence.

8. (a) Write True or False : 1

"The power series $\Sigma a_n z^n$ is absolutely convergent if the series $\Sigma|a_n z^n|$ is convergent."

(b) Find the pole of

$$f(z) = \frac{\sin(z-a)}{(z-a)^4}$$ 2

(c) Evaluate the following (any *two*) : 5×2=10

(i) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$

(ii) $\int_0^{\infty} \frac{dx}{1+x^2}$

(iii) $\int_{-\infty}^{\infty} \frac{\cos x\, dx}{(x^2+a^2)(x^2+b^2)}; \quad a>0,\ b>0$

(iv) $\int_0^{\infty} \frac{x \sin mx}{x^2+a^2} dx, \quad m \geq 0$

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